MATHS ANXIETY

Problems With Mathematics

OK, you thought you could get away with never having to open another maths textbook, and yet you're passionate about following some particular line of study and you've been told you now need some maths. The last time you did any maths was probably some

At least you're several steps ahead of the junior high school student who, not only doesn't understand maths, but really can't be bothered. That might have been you once! But now you're highly motivated. Like it or not you *need* maths to do advanced chemistry, or environmental science, or forensic science or something really interesting.

I'm assuming that you're a normally well-adjusted adult who is just scared of maths. Coming back to it after some years gives you two advantages. You're more motivated and you're more mature. You might still be *mathematically* immature but no doubt you're mature intellectually in other areas.

So why does mathematics have such a bad reputation? As a mathematician, I find that at social occasions, when it comes out that I teach mathematics, I nearly always get some outburst about how the other person hated maths, or was never very good at it. Yet if I hear my friend say they're a professor of psychology, or astronomy, or literature the response is usually "how interesting".

Many people boast that they "can't do maths", whereas this is not the case with speaking German, understanding political history or playing cricket.

The problem with maths, compared to practically every other area of study, is that it's linear. It builds on previous knowledge to an enormous extent. If you miss the first ten minutes of a history lecture you can usually pick up the thread. Miss the first ten minutes of a maths lesson and the chances are you'll be perplexed for the rest of the hour. Mathematics, as an area of study, is quite unique.

Mathematics is an Ancient Discipline

Somebody once said that mathematics is the second oldest profession! Certainly its roots go back much further than any of the sciences. Physics and chemistry go back only a few hundred years. Psychology is much younger. Mathematics began with prehistory. Moreover, nothing in mathematics has been superseded by later theories. Discoveries have been built upon previous ones so that the body of knowledge known as mathematics is the most mature of all areas of knowledge.

Probably almost all of what you know in mathematics was known in the middle ages. If you had done the highest level of school mathematics it would only have brought you up to the time of Elizabeth I. Doing a bachelor's degree in mathematics might, with a few exceptions, bring you up to the time of Queen Victoria. When people hear about research in mathematics they often reply, "wasn't it all worked out a long time ago?" It comes as a surprise that new mathematics is being discovered at an ever-increasing rate. *Mathematical Reviews* is a journal that lists short summaries of the more important mathematical papers that are published in one month. It's now on-line, but the last time paper copies were published, each month's issue was as big as a telephone book. Whole new branches of mathematics have sprung up in the last sixty years.

Mathematics as Storytelling

"What I like about mathematics", somebody once told me, "is that everything is black and white. You know where you stand. Mathematical truth is absolute." They were no doubt thinking of disciplines such as history, or psychology where there are conflicting schools of thought, and even facts that are disputed. Ask a physicist whether light consists of particles or waves and he or she will say, "well, it depends on how you look at it." Once, atoms were believed to be indivisible particles like billiard balls. Now physicists haven't got to the bottom of the sub-atomic particles from which they're formed.

In a sense, mathematics has no facts. Everything is relative. It says that *if* you make certain assumptions *then* such and such must logically follow. But there are no absolute truths in mathematics.

But surely "1 + 1 = 2" is an absolute fact! Certainly mathematicians long ago developed a system of numbers where 1 + 1 = 2, and this system seems to be useful in the real world when we want to count things. But mathematicians have also invented a system where 1 + 1 = 0, and this has proved extremely useful in computer science.

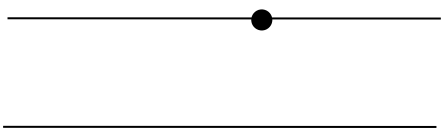
Mathematicians are the storytellers of the scientific world. They create great myths. Now myths can convey truth, even if the events are not true in the historical sense. Myths are about worlds other than our own material world and these worlds live inside the human imagination.

The same is true of mathematics. It lives only in the imagination. After all, mathematicians happily talk of a perfectly round circle or an infinitely long line. Neither exists in the real world. We might draw a pretty good circle on a piece of paper with geometrical instruments, but examine it under a microscope and you'll see that it's far from perfectly round. Moreover it will have thickness, which a mathematical circle does not. You can represent an infinitely long line on paper, but the dimensions of the page will limit how long it actually is.

Mathematicians create imaginary worlds. It's an intrinsically abstract intellectual pursuit. Somebody once said that if the universe suddenly disappeared the only areas of knowledge that would survive are mathematics and theology. Geology needs rocks, astronomy needs stars. Mathematics is independent of the material world.

Of course, it could be argued that if mathematics exists in the human mind it will disappear once the human race dies out. But then there are those who claim that God invented mathematics and He was there before creation. I'm not taking sides on this question! The point I'm making is that mathematicians makes up stories which scientists are free to use if they find it helps.

Take the question of the angles of a triangle. We believe that they always add up to 180 degrees. Well, Euclid stated some axioms from which Euclidean geometry can be developed, and in that geometry it can be proved mathematically that the angles of a triangle *always* total 180 degrees. These axioms were considered to be self-evident. But in the 1800s mathematicians questioned one of these axioms. This axiom is the so-called Parallel Postulate, that states that given any line, and a point that is not on that line, there is exactly one line that passes through the given point and is parallel to the given line.



These mathematicians wondered what would happen if there was more than one line through the point that is parallel to the given line, or perhaps no such line in some cases. By modifying the parallel postulate they developed new geometries where the angles of a triangle can be proved to total more than 180 degrees, or less than 180 degrees.

But surely, using careful measurement, we can prove that you always get 180 degrees. We might do that, but in so doing we would have stopped being a mathematician and started acting like an experimental physicist. One problem with experimental science is that you're limited by the accuracy of your instruments. Perhaps the angles of a triangle always add up to 179.999999 degrees. Perhaps the discrepancy will only show up if the sides of the triangle need to be measured in light years.

The other problem with experimental science is that we can only test the hypothesis a finite number of times. What if we could measure angles exactly and in every case that we tried the angles added up to exactly 180 degrees? That doesn't prove that it works for all the triangles we *haven't* tried.

Mathematicians have developed many geometries in the world of their imagination. They say to scientists, "here are some mathematical models for you to investigate". It's the role of the experimental scientist to decide which one seems to work best in the material world. And which one have they settled on? For most scientific purposes they're happy with the good old flat geometry of Euclid. But very close to an atom there is experimental evidence that space is "curved" and scientists have adopted one of the non-Euclidean geometries to describe that phenomenon.

Mathematics – Left Brain or Right Brain?

The human brain is divided into two hemispheres. The left brain controls the right side of the body, and vice versa. But psychologists have a theory that people with a dominant left hemisphere are strong logically while those with a dominant right hemisphere are more creative and imaginative. For this reason many people say "I can't do maths because I'm more right brain."

It's true that mathematics requires logical thinking. But it also requires imagination. After all, the concepts of mathematics are all imaginary. Take the number "3" for example. You've never held the number 3. Although you've seen the symbol "3" you've never seen the number 3 itself. The symbol "3" is just a name for the abstract entity 3. After all, just because you've seen the word "mermaid" doesn't mean you've seen an actual mermaid!

Mathematics is as much right brain as it is left brain. In some universities you can major in mathematics as part of an arts degree or as part of a science degree. Maths is *both* arts and science.

Yes, logical thinking is vital if you're going to study mathematics. But logical thinking is just the ability to follow rules. If you can find your way round a smart phone or play cards then you have no trouble with logical thinking. Logical thinking just requires being disciplined. If you're not prepared to follow the rules of a game, and constantly say, "well I *should* be allowed to trump that card even if the rules say I can't" then you'll have problems with mathematics. There are certain rules and you must follow them. Only when you get to be a research mathematician do you get to make up, or modify, the rules.

But being able to follow rules is not enough to give you the ability to do mathematics. I once thought that anybody could do mathematics provided they had a patient teacher who could explain the rules clearly. That was until my daughter did maths for the higher school certificate. I discovered that I could explain a question clearly, step by step, and she would understand it. But, when I gave her a very similar question to do, she was stumped. "I understood the previous question but you haven't shown me how to do *this* one." I would reply "can't you adapt what you learnt on the previous question to this one? The methods you need to use are essentially the same."

Now my daughter didn't lack creative thinking. It's just that she thought she had to leave creative thinking behind. Mathematics involves a combination of logical thinking and creative thinking.

It's often said that the reason why the modern generation is so bad at mathematics is because they're allowed to use calculators and they're not sufficiently drilled in their times tables. There's some truth in that ... but not much.

Let me make the bold claim that the drop in mathematical ability is because we no longer teach formal grammar! There's some truth in that, though probably not much. But what children used to get a lot of practice with was recognising similes and metaphors.

Metaphor is what mathematics is built on. Perhaps not the poetic kind, as in "the clouds closed across the moon like gossamer curtains", but metaphor is intrinsic to mathematics. The ability to see when a certain question is "essentially the same as one you've just seen" is vital.

Recognising that two things are different in some respects but the same in others is what makes the metaphor. Of course the sun is not a warrior in a chariot. But as it moves across the sky we can think of a chariot racing across our field of view. We mightn't think of it spontaneously, but when we read it in a poem we should be able to respond by saying "I see what he means." The person with a purely logical mind would say "don't be silly. A warrior and his horse couldn't live out there in space – there's no air!"

What exactly is the number 3? You may not have thought about it as being a very abstract concept. How did you first learn about three? Children learn to count before they have any notion of what the numbers mean. "One, two, three, four, five, six, seven, eight, nine, ten." What child has never felt pride in being able to count to ten? At this level counting is just a sequence of meaningless words. A parrot could be taught to do the same.

But a parrot has no concept of what those sounds mean. Nor does the little child when he or she first learns to count. But soon the child is shown a picture of three ducks, then three pigs and three elephants. She might form the idea that "three" has something to do with animals. But when she's shown three umbrellas, three balls and three houses she begins to abstract the three-ness from these collections. But, for some time, she might believe that you can only count things that are the same. Perhaps the balls have different colours but all three objects are balls. It may take some time before she can count the items in a picture of a house, a teddy bear and a horse.

Mathematics is full of analogy. Something in one branch of mathematics reminds us of something in another, and that analogy can be exploited. Problems in geometry can often be solved more easily by translating them into equivalent problems in algebra.

Overcoming Maths Anxiety

How can you deal with maths anxiety. One way that helps is to strengthen your basic skills. Mathematics requires a combination of logic and imagination, and each of these can be strengthened even before you open the pages of a mathematics book.

LOGIC

Logical thinking is simply the ability to mindlessly follow rules. Of course a computer can do that really well so you need to be able to put yourself into "computer mode". A modern trend in the teaching of mathematics is to emphasise *why* things work. To teach a mindless recipe for solving a certain type of problem is considered to be dreadfully old-fashioned. You're supposed to *understand* the steps.

This, of course, is to misunderstand the nature of mathematics. At the heart of mathematics is the ability to mindlessly follow certain rules. Trying to understand what you're doing, when you're first shown some sort of mathematical process, just makes life difficult. There's a time for teaching understanding but it should be when the student has acquired the ability to mindlessly perform the task.

The same is true of all learning. There's a lot to be said for rote learning. The problem with Victorian education is that they rarely got beyond it. Modern educational theories that emphasise creativity have added an important dimension to education, but it has come at the expense of children learning rules. Analysing why long division works is valuable but it should only come once a child has mastered the process as a mindless set of rules.

You probably know how to send an email. You mindlessly type out something like my.friend@webcompany.com.country. Do you understand the actual process by which your email gets to your friend's computer? Do you realise that it's sent from one computer to another, maybe through countries you've never even heard of. Do you know that your email might be chopped up into several bits that are sent separately, perhaps along different routes, until they're all reassembled at the other end? No, you just type out the email address mindlessly. If you become an IT person you may find it interesting, or even important, to know what lies behind the process. But you learn that you have to type the email address exactly, as if it was a meaningless string. If you tried to use the string my.dear.friend@webcompany.com.country your email would bounce back as being undeliverable.

In teaching mathematics to adults I've often encountered difficulties because they feel they should know *why* an algorithm works. If I try to explain the reasons it seems to confuse them in carrying out the process. They feel they can create their own rules that seem similar to mine. If I refuse to explain why, and ask them to "just do it" they feel I am treating them as children. It only works well when they allow themselves to accept the rules and acquire a technical ability to carry out the process (as a computer would) and *then*, once they can carry out the process easily, I can explain *why* it works.

Here's an exercise to try. On a sheet of paper carry out the following instructions:

(1) Write down the string of symbols x(+(x*))=)(x + 1
(2) Replace every x by (2y
(3) If you have a (followed by a), with no (or) between, delete the (and the) and everything in between
(4) Continue applying rule (3) until you cannot shorten the string any more
(5) Replace each y by))
(6) Apply rule (3) repeatedly until you can no longer shorten the string

What you've written after each step should be as follows:

(1) x(+(x*))=)(x+1
(2) (2y(+((2y*))=)((2y+1
(3) (2y(+()=)((2y+1
(4) (2y((2y+1
(5) (2))((2))+1
(6))+1

What does it all mean? Absolutely nothing! It was just a game with symbols, and not a very exciting one at that. Here's another task, for which you need to be standing up with your hands by your side. There are four instructions LEFT TURN, RIGHT TURN, LOAD and UNLOAD. LEFT TURN and RIGHT TURN are, as you might expect, instructions to rotate yourself. LOAD means to hold up an arm and to extend two fingers as if you are holding a gun. UNLOAD means to “fire” the gun, by which I mean that you drop your arm by your side. However, **whenever the gun is loaded you must do the opposite to what you are told**. If instructed to RIGHT TURN you turn to the left, and vice versa. If told to LOAD you must unload.

(1) Stand facing the door of your room with your arms by your side.
(2) LOAD
(3) LEFT TURN
(4) LOAD
(5) LEFT TURN

If you’ve done it correctly you should once again be standing facing the door. With your gun unloaded. Remember that at step (3) your gun is loaded so you must do a right turn. But at step (5) your gun is unloaded and so you obey the instruction and turn left.

Now repeat the instructions, but in a different order. Do (1), then (3), then (5), then (2) and then (4). This time your back will be to the door.

Does this mean anything? As a matter of fact, this time it does, but since you’re not learning group theory just treat it as another meaningless exercise in following rules.

IMAGINATION

Now following rules is only one discipline to learn. Imagination and intuition need to complement logic. To do maths you need to be able to recognise patterns and to develop a feeling for when two different things are essentially the same. This is the metaphor! Can you see any similarity in the following meaningless strings of symbols?

@@#$$$# and
99&!!!&

They’re different strings, involving different symbols. But in each case, the first symbol is repeated, the third and seventh are the same and the fourth fifth and sixth symbols are the same. They both follow the pattern AABCCCB.

You often come across puzzles where four things are shown and you have to find which is the “odd one out”. This means that you have to find some pattern that three of them share but which the other one doesn’t.

Which is the odd one out in the following list?

(a) @@#$$$#
(b) 77&!!!&
(c) 7$&&&7$
(d) 4473337

The intended answer is (c) because the others have the pattern AABCCCB.

But such questions are poorly stated. If you said that (d) is the odd one out because it is the only string that represents a number, I would be forced to accept that as a valid alternative. Or perhaps (a) is the odd one out because it is the only one that doesn’t include any digits.

Now I bet you skipped over these exercises because the weird symbols appear to have no meaning! Meaning is important, but to do mathematics well it’s important to be able to switch off and just look for patterns in meaningless strings of symbols. Look for metaphors and patterns in the world around you. Perhaps you might even read some poetry in preparation for doing mathematics!

Of course there's more to imagination and creativity than being able to construct or respond to metaphors. But creativity and imagination on its own is no good in mathematics. You'll never find a mathematics teacher praising a student's creativity in getting a wrong answer to an addition sum, saying "oh, that's an interesting answer". At its heart, mathematics is completely logical and unforgiving.

The difficult skill in doing mathematics is to be able to switch between mindlessly following rules and getting flashes of insight. Both are important for professional mathematicians, and both are important even for those just embarking on learning basic mathematics.