APPENDIX B PATTERNS OF PROOF

p	$\mathbf{p} \vee \mathbf{q}$	$\mathbf{p} \wedge \mathbf{q}$	$\mathbf{p} \rightarrow \mathbf{q}$
Suppose p	Suppose p		Suppose p
	is false	Hence p	
Contradiction!		•••••	
Hence p	Hence q	Hence q	Hence q

$p \rightarrow -q$	$-p \rightarrow -q$	$p \leftrightarrow q$
Suppose p	Suppose q	Suppose p
Suppose q		•••••
	•••••	Hence q
Contrad'n!	Hence p	Now suppose q
		Hence p

$p \vee q \rightarrow r$	$p \wedge q \rightarrow r$	$p \rightarrow q \vee r$
Case I	Suppose p	Suppose p
Suppose p	Suppose q	Suppose q
		is false
Hence r	Hence r	
Case II		Hence r
Suppose q		
Hence r		

$\exists x \in S [Px] \ \forall x \in S [Px] \ \forall x \in S [Px \rightarrow Qx]$

Let $x = \dots$	Let x∈S	Let x∈S
•••••	•••••	Suppose Px
•••••	•••••	•••••
Hence Px	Hence Px	Hence Qx

PROOF BY INDUCTION

ALL SIX STEPS MUST BE PRESENT

- (1) **CHECK** the first value
- (2) **SUPPOSE** result is true for *n*
- (3) **CONSIDER** the n + 1 case
- (4) **RELATE** it to the n case
- (5) **USE** induction hypothesis to prove the n+1 case
- (6) **CONCLUDE** proof by appealing to the induction principle

EXAMPLE: Prove $T_n = n(n-3)$ is even for all $n \ge 3$.

- (1) If n = 3, $T_n = 3 \times 0 = 0$ which is even. Hence the statement holds for n = 3.
- (2) Suppose the statement holds for n i.e. suppose T_n is even

(3)
$$T_{n+1} = (n+1)(n+1-3) = (n+1)(n-2) = n^2 - n - 2$$

(4)
$$= (n^2 - 3n) + 2n - 2 = T_n + 2(n-1)$$

- (5) Since T_n is even (by assumption)and 2(n-1) is even, then T_{n+1} is even.
- (6) Hence by induction, T_n is even for all $n \ge 3$.