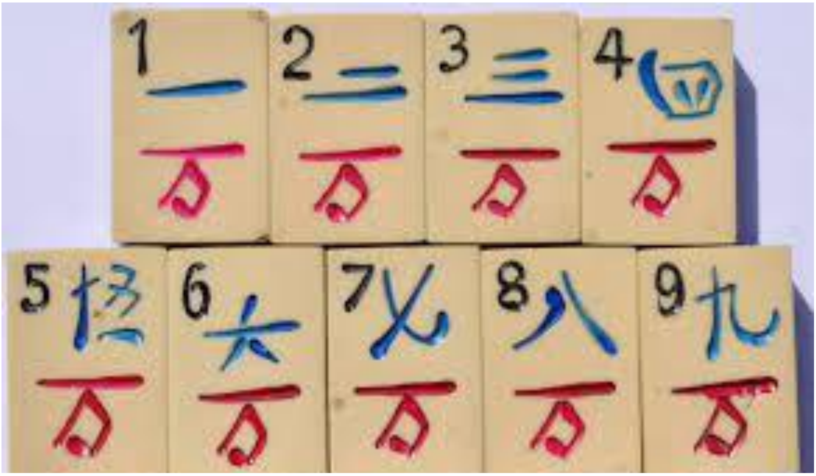
REPRESENTATION THEORY



CHARACTERS IN MAH JONG

Dr C. D. H. Cooper

Macquarie University

7th EDITION January 2022

These notes were prepared for students at Macquarie University in Australia but are freely available to anyone. However if you make use of them and are not a Macquarie University student it would be nice if you could email me at christopherdonaldcooper@gmail.com to let me know where you are from. And, if you are from outside of Australia perhaps you could send me a postcard of where you are from to pin up on my wall (Christopher Cooper, 31 Epping Avenue, EASTWOOD, NSW 2122, Australia).

INTRODUCTION

These notes assume that the reader has a good grounding in Group Theory as well as a firm grasp of Linear Algebra and Ring Theory. It is designed for students in their first year of postgraduate study. It is meant to accompany the notes on *Ring Theory*.

Linear Algebra is an important tool in Group Theory, because every finite group is isomorphic to a group of matrices.

A **representation** is a homomorphism from a finite group to a group of matrices. It turns out that homomorphisms, taken together, are more useful than isomorphisms.

We concentrate here on classical Representation Theory, which is over the field of complex numbers. This field has several useful properties which result in classical Representation Theory being more satisfactory, at least for a beginner, than over other fields.

For a start it's algebraically closed. Every matrix over $\mathbb{C}$ has its full complement of eigenvectors and is diagonalisable.

Secondly the field of complex numbers has characteristic zero. This means that the multiplicative identity, 1, has infinite additive order. In fields of 'finite characteristic', such as $\mathbb{Z}_p$, the prime characteristic gives problems when it divides the group order.

Finally, $\mathbb{C}$ has the operation of conjugation, where $x + iy$ maps to $x - iy$, and this is very useful.

Instead of studying the representations themselves, we mostly focus on the characters of representations. The character of a representation maps the matrix, or the linear transformation, to its trace. The trace of a square matrix is simply the sum of the diagonal elements and it is amazing, that with so much of the matrix thrown away, the trace can still contain so much information.

So we start with a group, map it to a group of matrices, and then map each matrix to its trace, so that a character is a map from the group to $\mathbb{C}$. Representations themselves are very difficult and tedious to compute but characters have so many nice properties that it is much easier to compute them directly, rather than by computing the representations first.

The so-called Fundamental Theorem of Characters lies at the heart of this body of information about characters. This is a deep theorem, but can be readily obtained from the Wedderburn Structure Theorem for semi-simple rings with descending chain condition. My *Ring Theory* notes develop this theorem and should be studied alongside these notes.

These notes contain a small amount of material that can't be found elsewhere, particularly in the chapter on Class Equations. They also contain many more examples of character tables than most texts on the subject.

CONTENTS

8. FROBENIUS GROUPS